3D Nonlinear Modeling of Magnetostrictive Materials Based on DEAM

Zhi Qin[1], Xiaoyu Xu[2], Shuai Yan[2], Hakeim Talleb[1], Zhuoxiang Ren[1]

[1]UPMC Univ. Paris 06, UR2, L2E, Paris F-75005, France
[2]Institute of Microelectronics of Chinese Academy of Science, Beijing 100029, China

This work presents the three-dimensional (3D) modeling of magnetostrictive materials. For the underlying dynamic magneto-elastic problem, mechanical displacements and vector magnetic potential are chosen as the working variables. Different field quantities are discretized with appropriate types of elements defined on nodes, edges, facets or volumes (Whitney elements). To account for the dependency of material constants on the stress and the magnetic field, the Discrete Energy-Averaged Model (DEAM) is incorporated into our finite element model (FEM), which, at the same time, involves a hierarchy of structures – with DEAM solved on the microscopic structure and FEM on the macroscopic structure. It also brings up nonlinearity that can be effectively resolved following a piece-wise linear approach. Representative numerical examples are also presented.

***Index Terms*—Magnetostrictive Materials, Finite Element Modeling, Whitney elements, Discrete Energy-Averaged Model**

I. Introduction

Magnetostrictive materials, thanks to their bi-coupled magneto-elastic features, are witnessing increasing interests in energy transmitting [1] and magnetic field sensing [2], to name a few. For the design of systems consisting of magnetostrictive materials, practical 3D magnetostrictive models are of critical importance. However, the development of such models is challenging: the underlying phenomenon is multiphysics, involving elasto- and magneto-dynamics on the macroscopic structure; additionally, material properties are nonlinear functions of stresses and magnetic fields, which requires calculating material constants with constitutive models on the microscopic structure. In this regard, most previous work found in the literature only addresses the issue partially.

On the macroscopic level, a considerable amount of magnetostrictive models, among the reported ones, are implemented via commercial packages like COMSOL Multiphysics. In this fashion, single field interfaces are predefined while couplings need to be established by the modeler. As introduced in [3], for example, the magnetostrictive coupling is introduced through specifying initial stresses and remanant flux densities as functions of respectively, magnetic field and mechanical strain. Obviously, implementing material constitutive models or any other un-predefined couplings is cumbersome, if ever feasible. Others, which do not rely on commercial packages, consist in enforcing the coupling in a weak manner; namely, magnetic and elastic problems are solved individually while couplings are enforced by adding additional magnetic or mechanical force terms [4]. The side effect of this approach is that data needs to be transferred between physics; iterations are also needed to assure equilibrium, which sometimes severely deteriorates the efficiency. Recently, a strongly coupled magnetostrictive model is presented in [5] where magnetic and elastic problems are solved simultaneously, based on the same mesh and eventually, resulting into a single block of discrete system, thus avoiding shortcomings of the former approach. Nonetheless, nodal elements are used for all quantities including the vector magnetic potential A, which is impractical from the implementing viewpoint. Moreover, the electrical potential ϕ is not considered in it, although it has been proved previously that in such cases the $A-\phi$ formulation is more suited in terms of numerical stability and convergence rates [6]. On the microscopic level, early magnetostrictive constitutive models, e.g. the modified Preisach model and the Jiles-Atherton model, consist in adapting ferromagnetic models to incorporate magnetostrictive couplings. As commented in [7], these models suffer from the insufficiency of accuracy. On the other hand, the more advanced magnetostrictive constitutive models consist in evaluating energy terms related to magnetocrystalline, magnetoelastic and magnetic fields along various orientations of magnetization. Then, the probability of a certain orientation is decided by the amount of the sum of the former energy terms, while the bulk magnetization and magnetostriction are obtained by considering all concerned orientations. Examples of such models are the modified Armstrong model [8] and DEAM [9], whose total number of considered orientations are respectively, 98 and six. In practice, DEAM is preferred due to its efficiency and ease to integrate with macroscopic models.

In this work, we build our 3D magnetostrictive model in a Whitney elements based framework, in which the advanced constitutive model is elaborated in a straightforward way. As such, dynamic elastic and magnetic fields are modeled in a unified way, thereby the magnetostrictive coupling is added naturally. Compared with previous 3D magnetostrictive models, ours is more advantageous in that it automatically preserves field properties on the continuous level after discretization. Also, it paves the way to integrate with 3D piezoelectric models (see e.g. [10]) when the magnetostrictive material is employed in two-phase magnetoelectric composites. Regarding the integration of FEM and DEAM, the former is resolved over all elements with material coefficients calculated using the latter, which is resolved over single elements and takes as inputs state variables extracted from the solution of FEM. The piece-wise linear approach is adopted in order to address nonlinearities involved during the process. In the rest of the digest, the FE formulations, implementation of DEAM, and some simulation results are presented. More details will be provided in a future extended version.

II. The 3D Multiphysics Modeling Process

A. Magnetostrictive constitutive equations

Constitutive equations of magnetostrictive materials [11], which state the relationship between magnetic and elastic quantities in the material, are expressed as in Eq. (1).

$$
\begin{aligned}
T_{ij} &= c^{B}_{ijkl}(T,H)S_{kl} - h_{kij}(T,H)B_k \\
H_i &= -h_{ikj}(T,H)S_{kl} + \nu^{S}_{ij}(T,H)B_k
\end{aligned}
\tag{1}
$$

where T indicates the stress, S strain, H the magnetic field, B magnetic induction; c^{B}_{ijkl} are elastic coefficients at constant magnetic field, ν^{S}_{ij} reluctivities at constant strain, and h_{kij} the magnetostrictive coupling coefficients.

B. The discrete energy-averaged model (DEAM)

As noted from above, material coefficients are dependent of state variables T and H. Here we implement DEAM for the calculation of the coefficients. Fig. 1 depicts the simulation results of Galfenol with our implementation, which is verified with experimental measurements found in [12].

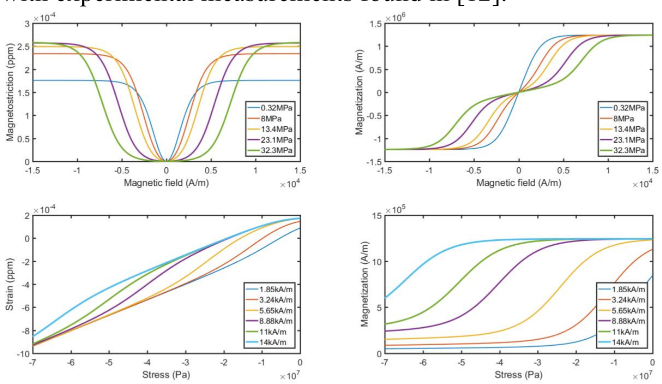

Fig. 1. Simulation results of nonhysteretic magnetostriction and magnetization for $\langle 100 \rangle Fe_{81.5}Ga_{18.5}$ at various stress and magnetic field levels.

C. Finite element discretization

The FE system is established by combining equilibrium equations (i.e. balance of linear momentum for elastodynamics, and the Maxwell's Equations for magnetodynamics) and the constitutive equations. Next, working variables are introduced before the weak form is obtained. Finally, quantities are interpolated with corresponding Whitney shape functions, which yields the discrete system after assembling. Our FE model can be applied to model magnetostrictive materials under various mechanical / magnetic conditions.

III. Numerical Examples

On top of the developed 3D model, we perform simulations of a magnetostrictive unimorph (i.e. cantilevered beam) whose operating scenario consists of superpositioning a harmonic magnetic excitation of small amplitude onto a properly biased point. As a result, simulations are carried out on two stages: we first solve a nonlinear static problem from which material coefficients at the biased condition are found; to calculate the dynamic response, a linear dynamic problem is resolved in which material coefficients are incremental and supposed to be constant and taken directly from the previous stage. The following two figures show part of the simulation results from both stages.

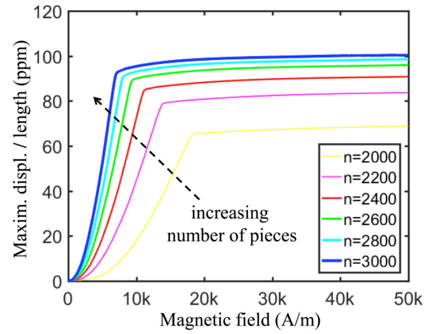

Fig. 2. Convergence study of the piece-wise linear approach (static).

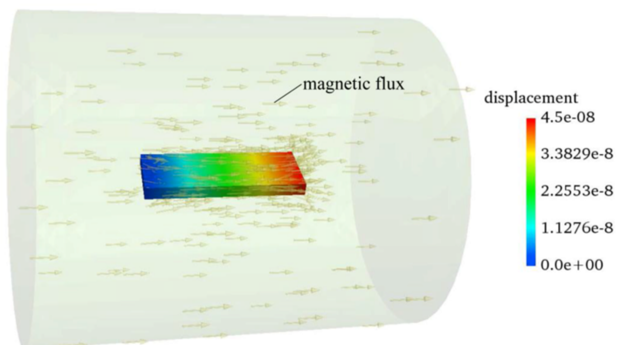

Fig. 3. Deformation (*scaled for representation purpose*) of a magnetostrictive unimorph under magnetic excitations in the length direction (dynamic).

References

[1] H. Talleb and Z. Ren, "Finite element modeling of magnetoelectric laminate composites in considering nonlinear and load effects for energy harvesting", *J. Alloys Compd.*, vol. 615, pp. 65-74, 2014.

[2] T.T. Nguyen, F. Bouillault, L. Daniel, and X. Minminger, "Finite element modeling of magnetic field sensors based on nonlinear magnetoelectric effect", *J. Appl. Phys.*, vol. 109, no. 8, pp. 84904, 2011

[3] "COMSOL Multiphysics", [online]. Available: www.comsol.com

[4] M. Schinnerl, M. Kaltenbacher, U. Langer, R. Lerch, and J. Schoeberl, "An efficient method for the numerical simulation of magneto-mechanical sensors and actuators", *Eur. J. Appl. Math.*, vol. 18, no. 2, pp. 233, 2007

[5] P. G. Evans and M. J. Dapino, "Dynamic model for 3D magnetostrictive transducers", *IEEE Trans. Magn.*, vol. 47, no. 1, pp. 221-230, 2011

[6] Z. Ren, and A. Razek, "Comparison of 3D eddy current formulations in dual systems", *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 751-755, 2000

[7] J. Atulasimha and A. B. Flatau, "A review of magnetostrictive iro-gallium alloys", *Smart Mater. Struct.*, vol. 20, no. 4, pp. 43001, 2011

[8] J. Atulasimha, "Characterization and modeling of the magnetomechanical behavior of Iron-Gallium alloys", PhD dissertation, Dept. Aero. Eng., Univ. Maryland, College Park, MD, US, 2006

[9] P. G. Evans and M. J. Dapino, "Efficient magnetic hysteresis model for field and stress application in magnetostrictive Galfennol", *J. Appl. Phys.*, vol. 107, no. 6, pp. 1-11, 2010

[10] Z. Qin, H. Talleb, S. Yan, X. Xu, and Z. Ren, "Application of PGD on parametric modeling of a piezoelectric energy harvester", *IEEE Trans. Magn.*, vol. 52, no. 11, pp. 7210211, 2016

[11] IEEE Standard Board, "IEEE Standard on Magnetostrictive Materials: Piezomagnetic Nomenclature", *IEEE Trans. Sonics Ultrason.*, vol. 20, pp. 67-77, 1973

[12] H. Tari, J. J. Scheidler, and M. J. Dapino, "Robust solution procedure for the discretec energy-averaged model on the calculation of 3D hysteretic magnetization and magnetostriction of iron-gallium alloys", *J. Magn. Magn. Mater.*, vol. 384, pp. 266-275, 2015